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ABSTRACT

There are numerous issues in the teaching of mathematics that need to be clarified and meaningfully understood by teachers, supervisors, and administrators. This paper discusses some of these issues, such as the integrated versus the separate subjects curriculum, the level of application versus knowledge for its own sake, state mandated versus locally determined objectives, traditional versus portfolio approaches in appraising achievement, traditional materials of teaching versus multimedia procedures, and logical versus psychological sequence in mathematics. One problem, among others in teaching mathematics, is how to arrange the order of objectives, learning opportunities, and evaluation techniques. There are numerous variables inherent in teaching pupils such as learning styles and multiple intelligences. (Contains 11 references.) (ASK)

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Issues in the Teaching of Mathematics

by Marlow Ediger

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ISSUES IN THE TEACHING OF MATHEMATICS

There are numerous issues in the teaching of mathematics. These issues need to be clarified and understood meaningfully by teachers, supervisors, and administrators. Too frequently, issues are vague and lack clarity and therefore are not meaningful to educators. Which issues need to be studied and synthesized, if possible?

The Integrated Versus the Separate Subjects Curriculum

How much of integration of subject matter should there be in the mathematics curriculum? Toward one end of the spectrum, there are educators advocating a separate subjects approach. Here, mathematics has its very own scope (what should be taught) as well as its own sequence (when should the content identified be taught)? Perhaps, mathematics has a more readily identifiable scope and sequence as compared to other academic areas. I believe that in the arithmetic curriculum all educators agree, with degrees of correlation and integration, that addition should be taught first, followed by subtraction, then multiplication, and followed by division in the primary grade sequence (Ediger, 1996). Increasingly at higher grade levels, the content can be become increasingly more complex as learners indicate readiness is in evidence. No educator would say that for primary grade pupils the sequence should be division, multiplication, subtraction, and then addition. Within the concept of addition beyond single digit addends, subject matter may well increase in complexity, such as having more than two single digits to add, having regrouping and renaming, and having more than one place value to add numerals.

Pupils may well work on correlated mathematics programs whereby addition and multiplication are involved in determining the perimeter of a geometrical figure, such as a rectangle (P= 2L+2W). A fused mathematics curriculum is involved when pupils related arithmetic, algebra, and statistics. Thus in statistics, pupils at a young age with teacher guidance may make a picture graph from everyday experiences such as birthdays that occur on each of the days of the month. Thus, for example, there may be three pictures for January, two for February, one for March, and so on, depending on when each pupil in the class had his/her birthday.

I believe problems arise in the integrated curriculum in mathematics when social studies, science, and reading/language arts are brought in heavily. Mathematics, however, has always been brought in to the curriculum for other academic areas.

In social studies, the following are examples of ways of integration

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that I have observed when supervising student teachers and cooperating teachers in the public schools:

1. determining how many years have elapsed since a certain event occurred such as the beginning of the Civil War in 1861.
2. learning about the Egyptian system of numeration when studying a unit on Egypt in the Middle East.
3. studying the abacus when a unit is being taught on the Philippines.

In science, much use is and can be made of mathematics including the following:

1. the umber of electrons, neutrons, and protons in an atom of a particular element.
2. the number of elements that go into the making of particular compounds.
3. the velocity of objects falling from different heights.

The language arts permeate all curriculum areas including mathematics, including the following mathematics/language arts correlations:

1. reading of word problems in mathematics.
2. listening to and speaking in committee or collaborative endeavors.
3. writing solutions to mathematics problems.

In the language arts/mathematics correlation, the listening, speaking, reading, and writing vocabularies are an inherent part of mathematics. One of five major objectives stressed by the National Council Teachers of Mathematics (NCTM, 1989) emphasizes learning to communicate ideas mathematically. Certainly, each pupil needs to become highly effective in communicating ideas to others.

An extreme case of the integrated mathematics curriculum might be the following when studying a thematic unit on fish:

1. pupils solving mathematics word problems on fish.
2. pupils studying fish in terms of scientific classification.
3. pupils singing songs of fish in music classes.
4. pupils doing art work about fish such as murals, dioramas, pencil sketching, water coloring, and model making.
5. pupils reading about fish using a variety of reference sources.
6. pupils engaging in physical education activities by doing motions and movements of diverse kinds of fish.

From the above, I believe we can all see that integration of content can go overboard. One does not want to lose sight of the scope and sequence inherent in mathematics. Mathematics is highly exacting and precise. It is good to relate ideas from other academic disciples if it assists pupils to understand mathematics better in terms of what is being

studied. This goes back to the statement of objectives. These objectives need to be carefully chosen so that pupils achieve what is essential and basic in mathematics.

The Level of Application Versus Knowledge for its Own Sake

Dr. John Dewey (1917) was an early advocate of knowledge being practical in that it can be used in society. His beliefs emphasized that knowledge which is used will be remembered better than knowledge acquired for its own sake. Also, if pupils can see how something is to be used, they will find purpose in learning. Reasons are then inherent in pupils wanting to learn. Aristotle (388-322 BC) was an early advocate of knowledge being learned for its own sake. Presently, Mortimer Adler (1902-) is an advocate of knowledge being valuable for its own sake. Thus, knowledge has its own value in and of itself (Ediger, 1995).

First, I will discuss knowledge that is learned and used in problem solving situations. Higher levels of cognition is necessary presently for pupils to function well, as possible as in the future workplace/society. Then to the level of application needs adequate emphasis since mathematics permeates our lives in everyday situations. A sixth grade class was asked by the student teacher and cooperating teacher, working as a team whom I supervised in the public schools, how mathematics is used in the daily lives of individuals. The length of the list was quite long, with thirty five different ways cited. I will mention ten of these ways:

1. writing checks
2. keeping a checkbook balance
3. paying from an invoice from Master Card.
4. counting change in one's pocket or billfold
5. paying cash for items purchased
6. giving money to charities
7. writing diary entrees and listing the dates pertaining thereto
8. using numerals to find a page in a book
9. crossing dates off on a calendar
10. keeping track of allowance money on hand as well as of that spent.

From the above, it is quite obvious that mathematics is very functional and important in the lives of individuals. Solving problems of living are generally involved in applying what has been learned since a new situation is involved each time that mathematics is used.
In problem solving, pupils with teacher guidance identify one or more problems. Each problem needs to be adequately delimited so that clarity

is involved. Information may then be gathered in answer to the problem. Higher levels of cognition need to be stressed such as critical and creative thinking. Pupils then appraise the content in terms of its worth and in terms of being valuable for the problem to be solved. Creative thinking also involves higher levels of cognition in that novel, new ways may need to be sought to solve problems in mathematics. Next, the information gathered may be used in developing an hypothesis or tentative answer to the identified problem(s). The hypothesis may be tested in a lifelike situation to determine its quality. If evidence warrants, a new hypothesis may need to be developed.
There are, indeed, many practical situations to test an hypothesis. Testing here does not mean a paper pencil test, be it norm referenced or criterion referenced, but rather reality is a societal kind of use (Meyer, 1949).

Second, knowledge for its own sake will be discussed. Those educators advocating knowledge for its own sake believe that content is learned for the sake of doing so (See O'Neill, 1981). Knowledge then has the following inherent characteristics:

1. possesses its own intrinsic goals and values
2. is appreciated for its form (structural ideas and patterns)
3. contains beauty in the many designs that may be drawn and created (geometric patterns)
4. provides a foundation or readiness to understand other more complex leanings in mathematics as well as perceive relationships with other academic disciplines
5. may provide enjoyment for participants
6. can be taught in an exciting and challenging way so all may benefit from instruction
7. uses inductive procedures in learning
8. makes for interest in learning due to fascinating topics discussed in mathematics
9. stresses rational thinking due to humans being rational creatures
10. emphasizes logical thinking as a major goal of instruction.

The above are excellent goals that do need to be incorporated into mathematics lessons and units of study. A major objective that may come about here is the level of application and problem solving.
I believe that pupils would find many uses for mathematics when working toward achieving the above named objectives. However separate objectives stressing the solving of problems, lifelike and realistic, need to be pinpointed (Ediger, 1997).

State Mandated Versus Locally Determined Objectives

Testing certainly is emphasized much in today's mathematics

curriculum. Certainly, too much testing takes away instructional time in teaching and learning. State mandated testing may take the form of tests developed under the supervision of the stated department of education. Those involved in developing these tests are external to the local classroom; they are not involved in teaching or even knowing the pupils who will be taking the test. Most state mandated tests are called Criterion Referenced Tests (CRTs) since they relate directly to objectives also written on the state level. Teachers then have access to these objectives for use in guiding instruction. At selected intervals, pupils are measured, as prescribed by the state, to determine how well these objectives have been achieved. The tests are usually offered once a year on the second, fourth, and eight grades, although this will vary from state to state. Sometimes, a state will require a norm referenced test to be given to pupils within a state. Norm referenced tests contain no objectives for teachers to use in gauging their instruction. Thus, norm referenced tests lack the validity that CRTs can possess. In either case, these tests are a one shot affair whereby pupils need to reveal when the tests are given how well they are doing in instruction. There are definite weaknesses here:

1. a one shot case does not allow for several variables such as not showing over a period of time how well the pupil is achieving.
2. the one shoot situation does not permit observing how well pupils do on a daily basis in each curriculum area.
3. the one shot case may completely determine how the lay public feels schools are doing in pupil achievement.
4. the one shot case stress verbal learnings only, such as reading and responding to multiple choice items. What about pupil achievement being revealed in other ways such as in music, art, drama, and committee work, among others?
5. the one shot case stresses reporting pupil progress numerically, such as the percentile rating or grade equivalent. What about qualitative information on pupils' progress such as in attitudes and in creative writing activities?
6. the one shot case does not permit pupils to show how they would use or apply knowledge acquired in a lifelike situation.
7. the one shot situation does not permit input from pupils in terms of how they wish to be appraised.

Evaluation specialists need to work on improving the quality of measurement and appraisal instruments so that more effective appraisal of pupil progress is in evidence. Weaknesses of testing need to be identified and overhauled (See Ediger, Teaching Reading and the Language Arts in the Elementary School on "Appraising Reading Achievement," 1997).

Locally determined objectives and evaluation procedures have been emphasized for years in the public schools. When I started

teaching in 1951, there were no state mandated tests. Each teacher devised his/her own tests to ascertain pupil achievement.

Presently, selected educators stress constructivism as a philosophy of instruction (Rowen and Bourne, 1994). Here, pupils and the teacher may select objectives for the former to achieve. Leaning opportunity to achieve the chosen objectives are also decided upon in the classroom. Appraisal procedures to ascertain what pupils have learned are chosen within the classroom setting. There are then no outside groups, such as state mandated tests and their supporters, who evaluate pupil achievement. Local determination in the classroom or with a team of teachers, decides upon objectives, learning activities, and evaluation approaches. What is missed by pupils is identified when using diverse appraisal procedures, and remedied through a variety of learning opportunities. Reasons provided to have curriculum development, including appraisal procedures, implemented on the local or classroom level are the following:

1. pupils and teachers are in the best position to know what has been taught and evaluation procedures are valid when alignment is there with the objectives of instruction.

2. tests written by outsiders, such as on the state level, may not harmonize with what has been taught in a specific classroom. Even more so, there are advocates of multiple intelligences whereby pupils individually possess unique talents in responding to additional ways in evaluation than paper/pencil tests. Verbal intelligence is used in responding to paper/pencil tests. Musical, artistic, kinesthetic, among others, are additional ways for pupils to indicate what has been learned in mathematics.

3. teachers on the local level have the best opportunities to study and learn about characteristics of children taught in mathematics. Educators on the state level do not know the children within any classroom; they are externally positioned to the teaching of pupils locally.

4. mathematics teachers should possess the knowledge and skills to sequence learning opportunities appropriately for pupils so that more optimal achievement is possible. Objectives determined on the state level provide no information on how to sequence these stated objectives.

5. it becomes a problem to align learning activities to be chosen by the teacher with the objectives selected by the state in the latters' mandated objectives.

Perhaps, the state mandated objectives versus locally determined objectives might be harmonized by having rational balance between the two approaches. There still is a problem in using state mandated objectives in that they are externally selected by those outside the classroom setting. Validity is then lacking in testing in that the evaluation procedures do not align properly with the objectives of instruction. An

exception would be if the teacher follows very carefully in choosing learning opportunities that assist pupils to achieve the state mandated objectives.

Traditional versus Portfolio Approaches in Appraising Achievement

Traditional procedures used in evaluating pupil achievement in mathematics have consisted of the use of norm referenced standardized tests as well as teacher written test items. Teacher written test items have consisted of solving word problems, testing on basic addition, subtraction, multiplication, and division facts, as well as using multiple choice, true/false, essay, matching, and completion items.These tests may measure pupil achievement in arithmetic, algebra, calculus, and statistics. Using teacher observation in daily work performed by pupils in mathematics will always be important. Here, the mathematics teacher needs to use updated criteria to appraise, diagnose, and remedy pupil errors and deficiencies.

A relatively recent innovation in appraising pupil achievement in mathematics emphasizes portfolio use (Isele, Frederick (1995). With portfolios, pupils are heavily involved in appraising their very own achievement. Ownership in evaluation, in large part, resides within the pupil. The teacher of mathematics is a guide and helper in the evaluation process. The portfolio in mathematics is a purposeful collection of pupil products and processes to show improvement over previous performances. The pupil with teacher guidance might then place the following of his work into the portfolio:

1. written daily work of pupils including problem solving activities, drill, and practice experiences. Test results of teacher written tests, including the written test items.
2. diagrams drawn to illustrate concepts understood in mathematics.
3. snapshots of construction objects completed in mathematics such as models of geometrical figures
4. videotapes of collaboration in mathematics, such as committee endeavors and large group work
5. individual activities completed by the learner involving projects.
6. journal entries written by the pupil in ongoing lessons and units of study in mathematics.
7. sequential daily diary entries to indicate what was learned on a specific day in an ongoing lesson.
8. logs written to combine diary entries and develop major generalizations and conclusions.
9. oral book reports in mathematics on a cassette.
10. justification for placing the above items in the mathematics portfolio.

The portfolio needs to be well organized with a preface and a table of contents. The portfolio is an excellent device to use in part/teacher conferences, as well as to indicate achievement in mathematics to interested, responsible persons such as the school administrator, supervisor, and curriculum director. In the portfolio, it is good to include items which show multiple intelligences in mathematics by revealing mathematics achievement of pupils through additional approach than verbal intelligence.

Traditional Materials of Teaching Versus Multimedia Procedures

Traditional methods of instruction have included use of basal textbooks, workbooks, worksheets, and some audiovisual materials. One problem inherent in using traditional approaches has been how these materials were used in teaching. The same materials used over and over again may make for boredom in teaching mathematics. Thus,if a mathematics text is used each day in teaching mathematics, with no other activities, it may make for a curriculum which lacks interest and challenge. A carefully chosen basal text, used with other materials of instruction, can develop increased interest in learning mathematics on the pupil's part. The mathematics teacher needs to study the pupil and his/her style of learning to notice what assists an individual pupil to achieve more optimally. In supervising student teachers and cooperating teachers in the public schools, I have noticed pupils who seemingly like a textbook centered mathematics curriculum, whereas others needed more semiconcrete or concrete set of experiences to achieve objectives in mathematics. Much, too, depends upon how the basal is used in teaching. A well selected textbook can definitely be used to have pupils think creatively and critically in solving word problems. Even in computations performed, mathematics teachers may guide pupils to think creatively by asking for different ways (algorithms) of computation, such as changing the order of addends or factors when using the commutative and associative properties of addition and multiplication respectively.

A textbook is a guide and can provide many quality learning opportunities for pupils. The basal then needs to have practical applications to every day life situations which the teacher needs to provide pupils. Whatever is learned must have use and application tendencies. If knowledge is not used, it is soon forgotten. To assist pupils in retention of mathematics knowledge and skills, content needs to be used in the many ways possible.

That brings up the problem of mathematics workbooks. Again with everyday use, pupils certainly can be turned off from these kinds of activities. I have noticed pupils working in workbooks with considerable interest. No doubt, there are pupils who prefer abstract sequential

experiences as compared to the semiconcrete and the concrete. Teachers who use mathematics workbooks in teaching pupils and have considerable success in doing so have a purpose in mind and they assist pupils to accept these purposes or reasons for the learning activity. Workbook activity needs to possess reasons for their use and these reasons (purposes) pupils need to accept intrinsically in order that achievement in mathematics is possible. It is always important to take a little time when initiating an activity in mathematics to discuss the reasons for these activities. The use of manipulative materials, illustrations, and audiovisual materials directly related to the ongoing lesson/unit can do much to guide more optimal achievement among pupils. Traditional procedures of instructions should

1. guide pupils to establish interest in the ongoing activities.
2. assist pupils to understand why the content is important to learn.
3. provide for quality sequence in the mathematics curriculum.
4. emphasize providing for individual learning styles and intelligences.
5. help pupils to achieve worthwhile objectives and omit trivia.
6. emphasize balance among cognitive, affective, and psychomotor objectives of instruction.
7. establish meaning and understanding of mathematical content among learners.
8. make provision for individual as well as collaborative learning activities.
9. keep pupils on task in a reasonable way.
10. secure pupil input into the mathematics curriculum.

Toward the other end of the continuum, a multimedia approach in teaching mathematics is advocated. Computer programs, videotapes, slides, filmstrips, films, internet, E-mail, and CD ROMS, among other technology, would provide a variety of activities for pupils. The state of the art technology might then be available to learners in a modern mathematics curriculum. Pupils with teacher guidance may then select and identify problems, gather data or answers to the mathematical problems, evaluate the acquired content critically and creatively, achieve an hypothesis or answer which is tentative, test the hypothesis in a virtual reality or lifelike situation, and make necessary modifications if necessary.

A multimedia approach in teaching mathematics provides opportunities for pupils to experience learning activities harmonizing with criteria such as the following:

1. emphasize diverse activities to develop and maintain interest in mathematics.
2. have pupils experience activities which harmonize with talents and abilities possessed.

3. provide a hands on approach to learning in mathematics.
4. guide pupils to attain vital objectives.
5. stress empowerment of pupils in mathematics.
6. assist pupils to reflect upon mathematical content acquired.
7. facilitate pupil ownership in the mathematics curriculum.
8. indicate deficiencies possessed by pupils with numerous opportunities to take care of these as necessary.
9. teach so that success in learning is in evidence.
10. teaching as facilitating learning in ongoing lessons and units in mathematics.

There needs to be room for both traditional and multimedia approaches in teaching and learning. Certainly, a carefully chosen basal has much to offer in providing relevancy, quality scope, and proper sequence in the mathematics curriculum. The teacher must assist pupil learning in mathematics by using recommended criteria from the psychology of education. Updating the mathematics curriculum strongly stresses a multimedia approach in learning. New procedures should not be accepted in teaching for the sake of doing so, but rather to help each pupil reach his/her potential in the mathematics curriculum.
The state of the art technology must assist teachers to plan the best objectives. learning opportunities, and evaluation procedures possible. Presently, mathematics is vital for each pupil as he/she progresses through the diverse levels of schooling as well as later on in the work place.

Logical Versus Psychological Sequence in Mathematics

There are mathematics educators who believe in a logical sequence in developing the mathematics curriculum. Here, the objectives need to be stated in measurable terms. The teacher determines which objectives to emphasize, written very precisely. Leaning opportunities then need to be selected by the teacher and arranged sequentially for pupils to achieve stated objectives in mathematics. Evaluation techniques need to be carefully aligned with the specific objectives. The evaluation procedures are used to determine which objectives have been achieved by each pupil. Ideally those not achieved should receive additional learning activities so that pupils may be successful achievers.

The measurably stated objectives, the learning opportunities, and the evaluation procedures are all carefully arranged so that the learning activities guide pupil to achieve each measurably stated objective in mathematics. The evaluation procedures measure if the learning activities assisted pupils to achieve the stated objectives.
The mathematics teacher in a logical manner arranges these three parts

of the curriculum so that optimal achievement is possible from each pupil.

Toward the other end of the mathematics curriculum, the pupils with teacher guidance have considerable input into the curriculum. A learning stations approach may be in evidence. Here, there may be seven stations, as an example, for a class of twenty-two pupils. Each station is labeled as to the title of the topic(s) to be covered. Concrete, semiconcrete, and abstract materials should be located at each station. Pupils individually or in committees may select the station to work at. Each station may have a card listing four to five things for pupils to engage in. The pupil is the chooser as to which tasks to complete. Each learner then sequences his/her own learning activities. The mathematics teacher becomes a guide and stimulates pupils to do well in mathematics. Time on task is important in any philosophy of teaching mathematics. There are more tasks as compared to what any pupil can complete so that the learner may omit those that lack perceived purpose. A psychological sequence is involved when the pupil, basically, sequences his/her own learning activities in mathematics (Ediger. 1988).

I believe there is room for both a logical and a psychological sequence in mathematics. Certainly, there are things that are essential for all pupils to learn such as addition, subtraction, multiplication, sand division within a problem solving framework The essentials will be teacher determined. The mathematics teacher than will sequence or order the arrangement of objectives, learning opportunities, and evaluation procedures for pupils.

I believe too there needs to be input from pupils to increasingly develop a child centered mathematics curriculum. Pupils have questions for which they would like to have answers. These questions may become problems to stress in a problem centered mathematics curriculum. The learning activities to sequence pupil learning may involve enrichment centers for pupil active engagement whereby personal choices may be made as to what to learn and the order of learning.

Conclusion

Mathematics teachers are concerned about having pupils achieve more optimally in mathematics. One problem, among others in teaching mathematics, is how to arrange the order of objectives, learning opportunities, and evaluation techniques. Should these be arranged in ascending order of complexity by the teacher? Or, should there be heavy pupil involvement in sequencing his/her own learning in mathematics?

There are numerous variables inherent in teaching pupils such as learning styles and multiple intelligences. Needs of pupils should be

met to improve the mathematics curriculum. Only then might pupils achieve more optimally!

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